GENERATION OF RANDOM TIME-SERIES THROUGH HYBRID COMPUTATION

Technical Report No. 35

PROJECT THEMIS

Systems Research Center
Industrial & Systems Engineering Department
University of Florida
Gainesville

Best
Available
Copy

GENERATION OF RANDOM TIME-SERIES THROUGH HYBRID COMPUTATION

Technical Report No. 35

SUBMITTED BY:

P. E. Valisalo (Co-Author A. H. Haddad)
Research Associate

APPROVED BY:

B. D. Sivazlian
Director, Project THEMIS

Department of Industrial and Systems Engineering
The University of Florida
Gainesville, Florida 32601

May 1970

This research was performed under Project THEMIS, ARO-D Contract No. DAH C04 68C0002.

Abstract

In many engineering design problems it is possible to collect data of the environmental disturbances which are acting upon our systems. This data can be analyzed by determining its autocorrelation and probability density function. When seeking solutions through simulation it is desirable to be able to generate random time series having a predetermined autocorrelation and probability density. This paper describes a method to control both simultaneously.

The proposed system is composed of a linear filter, H(s), with white Gaussian noise as input, followed by a nonlinear element f(x), where x(t) is the output of H(s). The output of the system y(t) is required to have a predetermined probability density $p_y(y)$ and a predetermined normalized autocorrelation function $\rho_y(\tau)$. The non-linearity f(x) is designed to give the required density $p_y(y)$, and is relatively easy to design using the relationship

$$f(x) = F_y^{-1}[F_x(x)]$$

where F_y^{-1} is the inverse of $F_y(y)$ which is the cumulative distribution function of y, and

$$F_x(x) = \int_{-\infty}^{x} \frac{e^{-\frac{\xi^2}{2}}}{\sqrt{2\pi}}\, d\xi$$

is the cumulative Gaussian distribution.

The nonlinearity $f(x)$ may be designed manually or in a digital computer by using a double table look up or in a hybrid computer system. It can then be stored in a variable diode function generator of an analog computer.

This nonlinear element, however, changes the autocorrelation of the input, $x(t)$. The amount of change can be determined by

$$\rho_y(\tau) = \sum_{n=1}^{N} \frac{K_n^2}{n!} [\rho_x(\tau)]^n$$

where

$$K_n = \int_{-\infty}^{\infty} f(x) H_n(x) \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$$ and

$H_n(x)$ is the Hermite polynomial of order n.

This calculation can be done most conveniently in a hybrid computer system.

The normalized autocorrelation of the intermediate stage, $x(t)$, $\rho_x(\tau)$ must now be approximated by a sum of exponentials:

$$\rho_x(\tau) = a_1 e^{-s_1|\tau|} + a_2 e^{-s_2|\tau|} + \ldots .$$

by using a hybrid computer system. One approach which may be used to determine the a_i's and s_i's is discussed by McDonough and Huggins. The filter $H(s)$ may be obtained from the a_i's and s_i's by spectral factorization.

By forcing the system with white Gaussian noise the system output, $y(t)$ will have the desired autocorrelation function and probability density.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. Introduction

Most of our physical systems are subjected to environmental disturbances. If we are supposed to improve our system through simulation then we must have means to generate disturbances as they appear in nature.

In most cases it is possible to record these random disturbances for further analysis. One way to analyze them is to determine their autocorrelation function and their probability density. Our task now is to design a system, which will generate a random time series having a predetermined autocorrelation and probability density. In other words we must be able to control both simultaneously.

Consider a system described in Figure 1. This system should be so designed that a white Gaussian noise as an input will give y(t) as an output having the desired normalized autocorrelation, $\rho_y(\tau)$ and the probability density, $p_y(y)$. Working backwards from y(t), the instantaneous nonlinearity, f(x) must be such as to guarantee the desired probability density, $p_y(y)$ with an input, x(t). The function x(t) is always Gaussian, but it must have a normalized autocorrelation, $\rho_x(\tau)$ such as to give the required autocorrelation, $\rho_y(\tau)$ after going through the nonlinear element f(x). The linear filter, H(s) must be so designed as to give Gaussian output, x(t) with zero mean and the required autocorrelation $\rho_x(\tau)$. For simplicity, the variance of x(t) will be normalized to unity. The nonlinear element, f(x) can be designed first because $p_y(y)$ and $p_x(x)$ are known. Then $\rho_x(\tau)$ can be determined because $\rho_y(\tau)$ and f(x) are known. $\rho_x(\tau)$ can now be approximated by a sum of exponentials. And finally, the

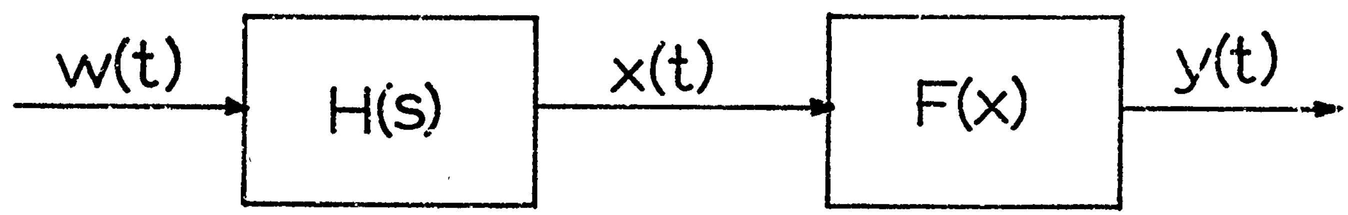


Figure 1. A system to generate random time-series having a predetermined autocorrelation and probability density.

linear filter may be obtained from the a_i's and s_i's by spectral factorization. We will now have a system which will continuously generate random time series having the required properties.

2. Mathematical Derivations.

The nonlinear element can be designed by using the relationship[3,5,6,7]

$$f(x) = F_y^{-1}[F_x(x)]$$

where:

F_y^{-1} is the inverse of $F_y(y)$

$F_y(y)$ is the integral of the desired distribution density, $p_y(y)$.

$F_x(x)$ is the integral of the Gaussian distribution density, $p_x(x)$ or

$$F_x(x) = \int_{-\infty}^{x} \frac{e^{-\frac{\xi^2}{2}}}{\sqrt{2\pi}} d\xi$$

Both cumulative density functions must be normalized so that the final values of the integrals are one.

The relationship of the normalized autocorrelations $\rho_x(\tau)$ and $\rho_y(\tau)$ can be determined from the approximate equation

$$\sigma_y^2 \rho_y(\tau) = \sum_{n=1}^{N} \frac{K_n^2}{n!} [\rho_x(\tau)]^n$$

where σ_y^2 is the variance of $y(t)$ (obtainable from $p_y(y)$),

$$K_n = \int_{-\infty}^{\infty} f(x)H_n(x) \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx \qquad \text{and}$$

$H_n(x)$ is the Hermite polynomial of order n[8]. The number of terms N, is determined by the desired normalized error, ε, in the approximation which is given by

$$\varepsilon = \left[1 - \sum_{n=1}^{N} \frac{K_n^2}{\sigma_y^2 n!}\right]$$

Since $\rho_y(\tau)$ and $f(x)$ are known, $\rho_x(\tau)$ can be derived. To design the filter, H(s) the normalized autocorrelation function of the output, $\rho_x(\tau)$ must be approximated to the desired level of accuracy. This can be done by finite sums of exponentials [19] as described by R. N. McDonough and W. H. Huggins [1]. In their approach, "... a given number n of exponentials is considered, such that the integrated squared error is minimized over both the n coefficients of the linear combination and the n exponents used." [from the abstract of ref. 1.].

If a hybrid computer, however, is available, then the mechanization of finding the exponents and coefficients can be simplified as discussed later in this paper. Also it is believed by these authors, that for most engineering cases only three exponentials are required.

The problem reduces now to finding the best estimate of $\rho_x(\tau)$, $\hat{\rho}_x(\tau)$ of the form:

$$\hat{\rho}_x(\tau) = a_1 e^{-s_1\tau} + a_2 e^{-s_2\tau} + a_3 e^{-s_3\tau} \qquad , \quad \tau \geq 0$$

These three exponents can be easily generated in an analog computer and by using optimizing hybrid techniques the parameters a_i's and s_i's can be so adjusted as to give the minimum integrated squared error. Then the power spectrum would be:

$$\Phi(s) = \frac{2a_1s_1}{s_1^2-s^2} + \frac{2a_2s_2}{s_2^2-s^2} + \frac{2a_3s_3}{s_3^2-s^2}$$

The filter H(s) may now be derived by spectral factorization which for the three exponents case is given as follows: [2, 4]

$$As^4 - 2Bs^2 + C$$

$$s^2 = \frac{B \pm \sqrt{B^2 - AC}}{A}$$

$$p_1 = \sqrt{\frac{B + \sqrt{B^2 - AC}}{A}}$$

$$p_2 = \sqrt{\frac{B - \sqrt{B^2 - AC}}{A}} \qquad \text{where}$$

$$A = 2(a_1s_1 + a_2s_2 + a_3s_3)$$

$$B = a_1s_1s_2^2 + a_1s_1s_3^2 + a_2s_2s_1^2 + a_2s_2s_3^2 + a_3s_3s_1^2 + a_3s_3s_2^2$$

$$C = 2(a_1s_1s_2^2s_3^2 + a_2s_2s_1^2s_3^2 + a_3s_3s_1^2s_2^2)$$

$$H(s) = \frac{[\sqrt{2a_1s_1 + 2a_2s_2 + 2a_3s_3}](s + p_1)\ (s + p_2)}{(s + s_1)\ (s + s_2)\ (s + s_3)}$$

or more conveniently, defining

$$\alpha = \sqrt{2a_1s_1 + 2a_2s_2 + 2a_3s_3}$$

$$H(s) = \frac{\alpha\ (p_1-s_1)\ (p_2-s_1)}{(s+s_1)(s_2-s_1)(s_3-s_1)} + \frac{\alpha(p_1-s_2)\ (p_2-s_2)}{(s+s_2)(s_1-s_2)(s_3-s_2)} + \frac{\alpha(p_1-s_3)(p_2-s_3)}{(s+s_3)(s_1-s_3)(s_2-s_3)}$$

In the general case with M exponents, we have

$$\hat{\rho}_x(\tau) = \sum_{i=1}^{M} a_i\ e^{-s_i\tau} \qquad , \quad \tau \geq 0$$

Then the power spectrum may be written in the form

$$\Phi(s) = \sum_{i=1}^{M} \frac{2a_i\, s_i}{(s_i^2 - s^2)} = \frac{\prod_{i=1}^{M-1} (p_i^2 - s^2)}{\prod_{i=1}^{M} (s_i^2 - s^2)}$$

and therefore, the filter is given by

$$H(s) = \frac{\prod_{i=1}^{M-1} (p_i + s)}{\prod_{i=1}^{M} (s_i + s)}$$

It is easy to see, that to build the filter the same analog computer circuit, which was used to find the a_i's and s_i's, can be used just by changing the setting of the three input potentiometers.

To clarify the method an example with an analytical solution is given in Appendix No. 1.

3. Computerized Methods.

When seeking solutions for these types of problems through hybrid computation the following steps have to be taken. First of all, two distinctively different stages can be defined, namely: the design stage and the stage of utilization of the designed system to generate random time series.

A. Design Stage.

1) The desired probability density function $p_y(y)$ is given either in a numerical or in a graph form. In the first case quite often a digital computer subroutine is available. In many cases it can be conveniently generated in an analog computer. In the second case a curve follower or magnetic ink could be used. In any event, the given $p_y(y)$ must be generated, digitalized, stored and plotted. It must be repetitively available for further study.

2) Gaussian probability density function in most cases is available as a digital computer subroutine. It must be generated and stored as in No. 1.

3) The stored $p_y(y)$ must be fed into analog computer for integration to form $F_y(y)$. This will then be sampled and stored in a digital computer.

4) Gaussian density function, $p_x(x)$ is integrated and stored as in No. 3 to form $F_x(x)$.

5) The given autocorrelation function of the output, y(t) in its normalized form $\rho_y(\tau)$ must be generated, digitalized, stored and plotted. Notice step No. 1.

6) Because $F_y(y)$ and $F_x(x)$ now are available the non-linear element, f(x) can be designed and stored in a digital computer. This can be done by using a double-look-up method in a digital computer as illustrated in Figure 2. In the final runs either a digital computer or a variable diode function generator in an analog computer could be used to implement the required non-linear element, f(x).

7) To generate K_n's, n = 0,1,...,N, we need f(x), $p_x(x)$ and the Hermite polynomial, $H_n(x)$. This can be done most conveniently in an analog computer as of Figure 3. Notice that two previous stages of $f_i(x)$ must be stored for the next set of K's. This should be continued until the error ε is satisfactorily small. [Hermite polynomials - $H_{n+1}(x) = xH_n(x) - nH_{n-1}(x)$; $H_0 = 1$, $H_1 = x$, $H_2 = x^2 - 1$, $H_3 = x^3 - 3x$.] K_0 is left in this circuit for checking purposes, since K_0 is equal to the mean of y, i.e. $K_0 = \int_{-\infty}^{\infty} yp_y(y)dy$.

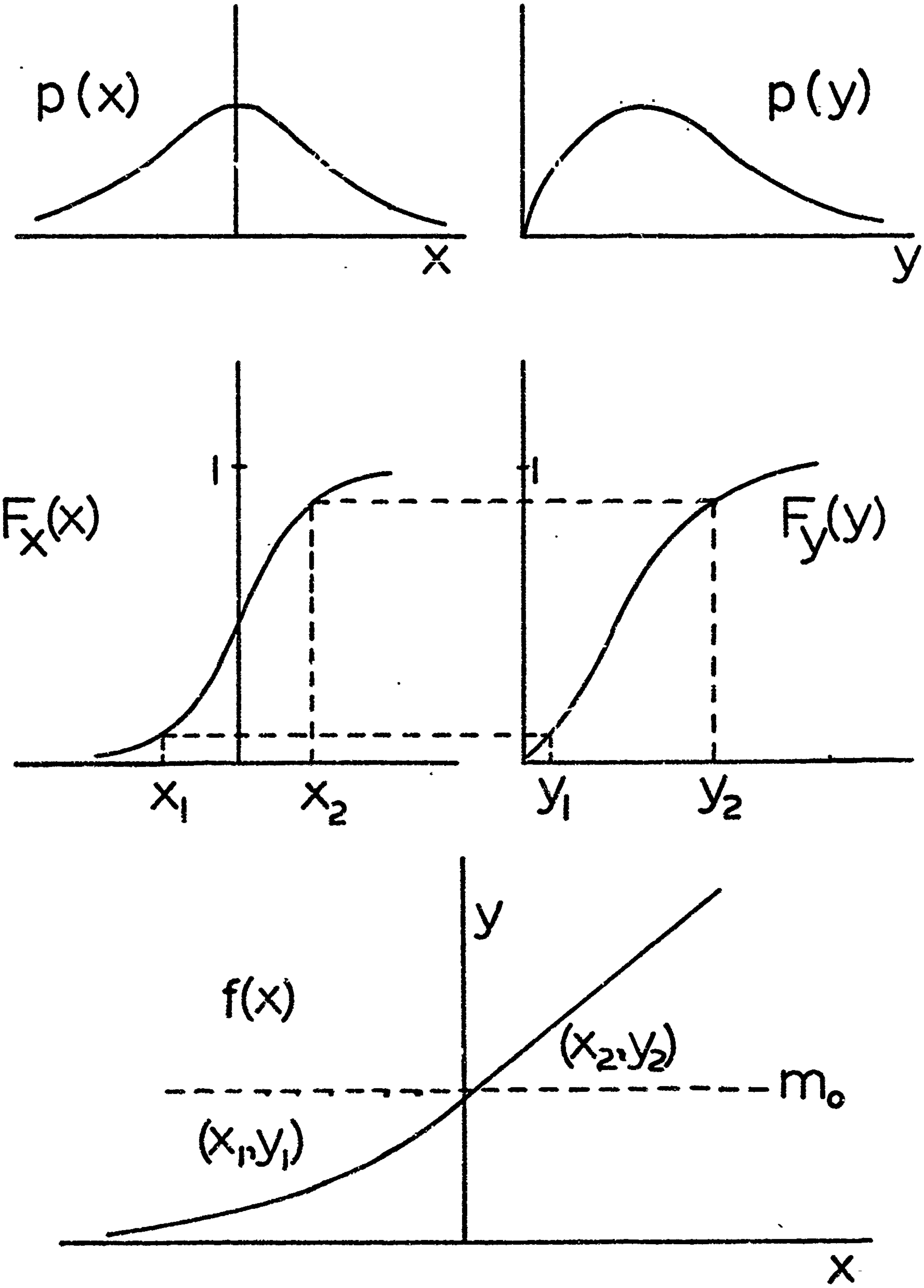


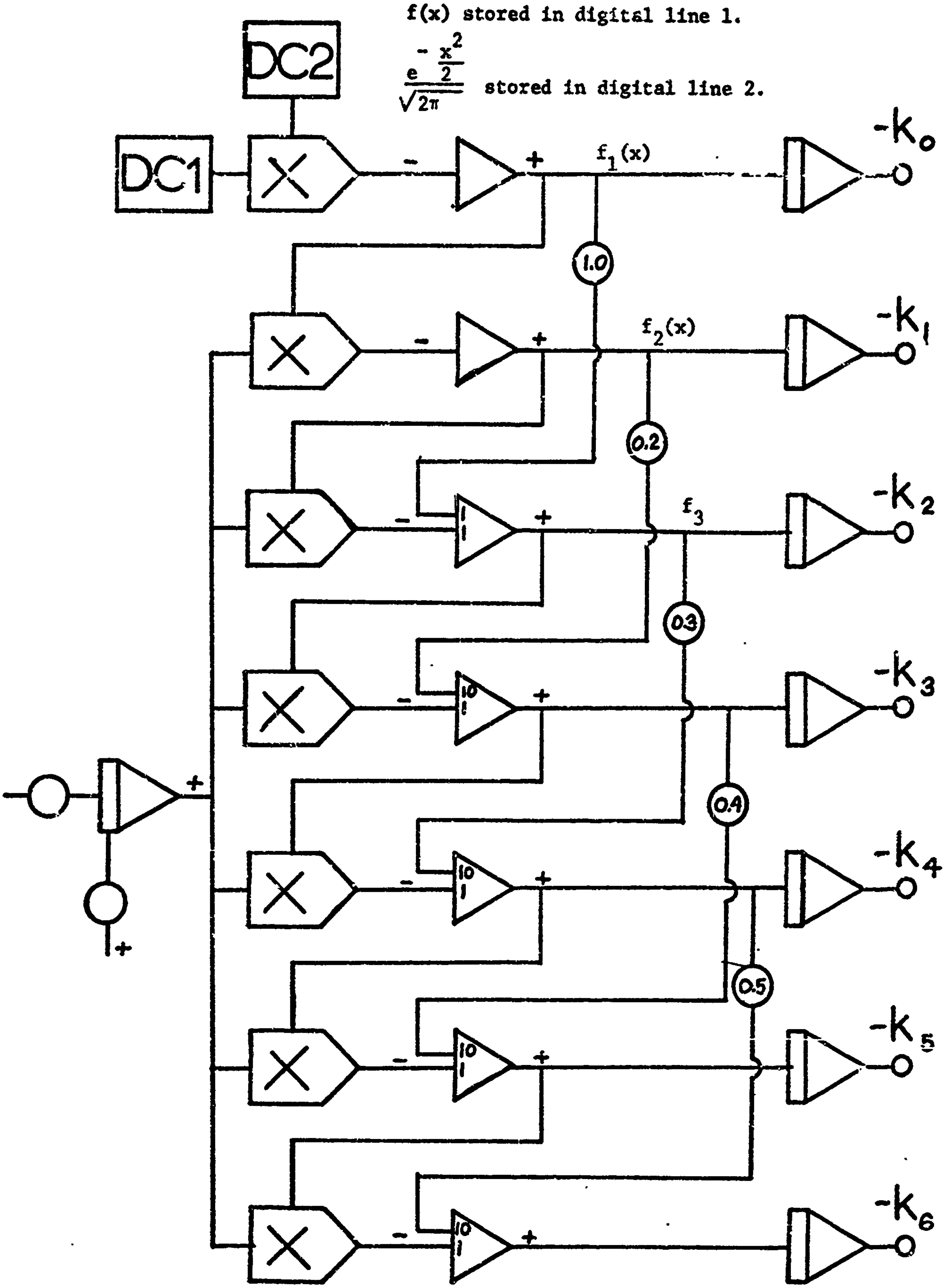
Figure 2. A demonstration of a double look-up method to design the non-linear element, f(x).

Figure 3. An analog computer circuit to generate K_i's.

$$K_n = \int_{-\infty}^{\infty} f(x)\, H_n(x)\, \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\, dx$$

f(x) stored in digital line 1.

$\dfrac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$ stored in digital line 2.



8) The normalized autocorrelation function of $x(t)$, $\rho_x(\tau)$ can now be generated, digitalized, stored and plotted. An illustration of how this can be done in a case where four values of K_i were found to be satisfactory, see Figure 4. $\rho_x(\tau)$ can be generated in C by letting a servo-multiplyer, A, maintain the output, B, at zero.

9) To approximate $\rho_x(\tau)$ by a sum of exponentials the following method is used as a first trial. Build exponential generating circuits in an analog computer, for example three of them. Take the absolute value of error between the sum of the exponents and the stored value of $\rho_x(\tau)$ and integrate in an analog computer. By using digital computer optimizing techniques readjust the servoset potentiometers representing the a_i's and s_i's as to minimize the error. Print out the optimizing values of a_i's and s_i's.

10) Design the filter H(s) as per Chapter 2. Notice that the analog computer circuit in Step 9 can be used by merely changing the values of the input potentiometers (and of course by removing the initial conditions).

B. Stage of Generation of Random Time Series

1) Apply white Gaussian noise to the system. Plot $\hat{\rho}_y(\tau)$ and compare with the given $\rho_y(\tau)$.

2) Plot $\hat{p}_y(y)$ and compare with the given $p_y(y)$.

3) Plot the impulse response of the filter, H(s).

4) Plot $\rho_x(\tau)$.

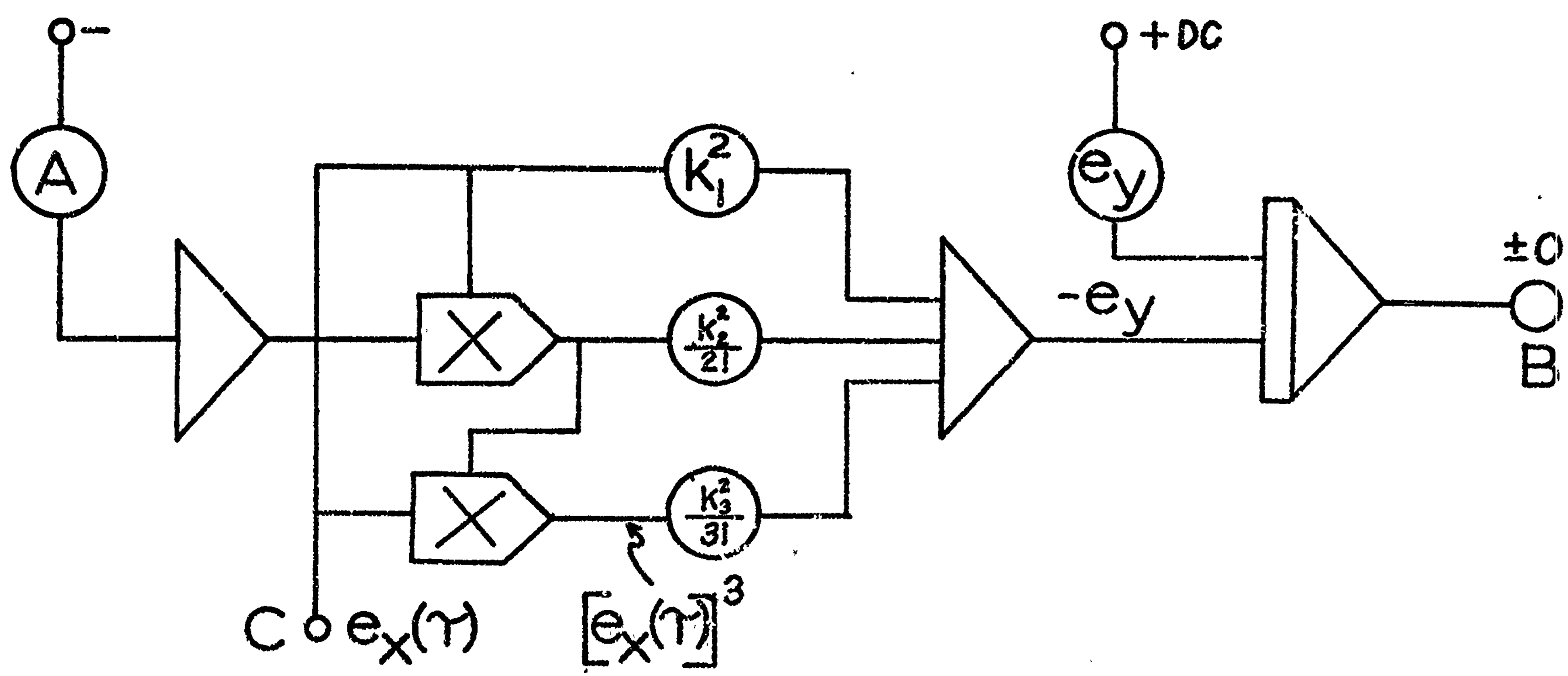


Figure 4. An analog computer circuit to generate $\rho_x(\tau)$. [Three terms].

$$\rho_y(\tau) = K_1^2\,\rho_x(\tau) + \frac{K_2^2}{2!}\,\rho_x^2(\tau) + \frac{K_3^2}{3!}\,\rho_x^3(\tau)$$

4. Experimental Computer Run.

For this experimentation an example, which has been analytically solved in Appendix 1., was chosen. This example, however, was originally designed for a fast hybrid system with a large memory. To fit the system available, the hybrid computer system of the Department of Industrial and Systems Engineering, University of Florida, Gainesville, Florida [10.] the time constant had to be increased. The required autocorrelation and probability density of the output, y(t) were

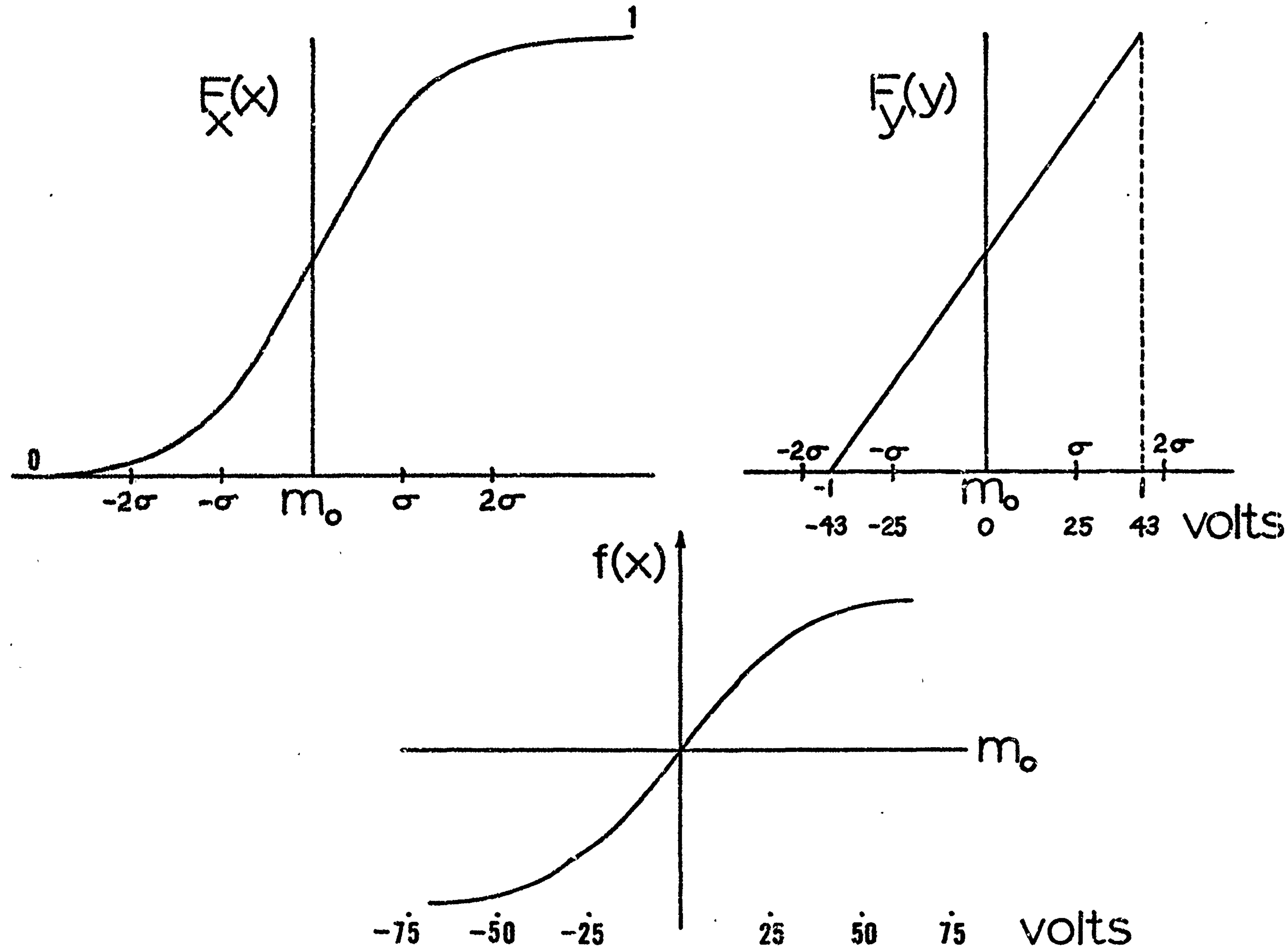
$$\rho_y(\tau) = e^{-\beta|\tau|} \quad \text{where} \quad \beta = .5$$

$$p_y(y) = \text{uniform between} - 1 \text{ and } + 1.$$

The linear filter, H(s) changes now to

$$H(s) = \frac{1.06}{s+.5} - \frac{.119}{s+1.5}$$

and Figure 5.a shows, how the non-linear element, f(x) was designed. This was stored in the variable diode function generator. A permanently stored subroutine was used to generate random-numbers having a uniform distribution [Fig. 5.b.]. Sequentially eight of these were averaged to give one value for the required normally distributed data [Fig. 5.c.]. As one can easily see there is a long range fluctuation present in this data. This irregular fluctuation could be called a DC-component and its effect is detrimental for the correllogram when one is forced to use such a short record of data, only 630 data points, which means that the base of an autocorrelation function can be only 315 data points. To remove at least part of this DC-component a double, exponential smoothing was used. This means that the data [Fig. 5.c.] was run through an exponential smoothing circuit with a time constant of 1 sec., then the smoothed data was run again



F_x(x)
1
0
-2σ
-σ
m_o
σ
2σ
F_y(y)
-2σ
-σ
σ
2σ
-1
m_o
1
-43
-25
0
25
43
volts
f(x)
m_o
-75
-50
-25
25
50
75
volts

Figure 5.a

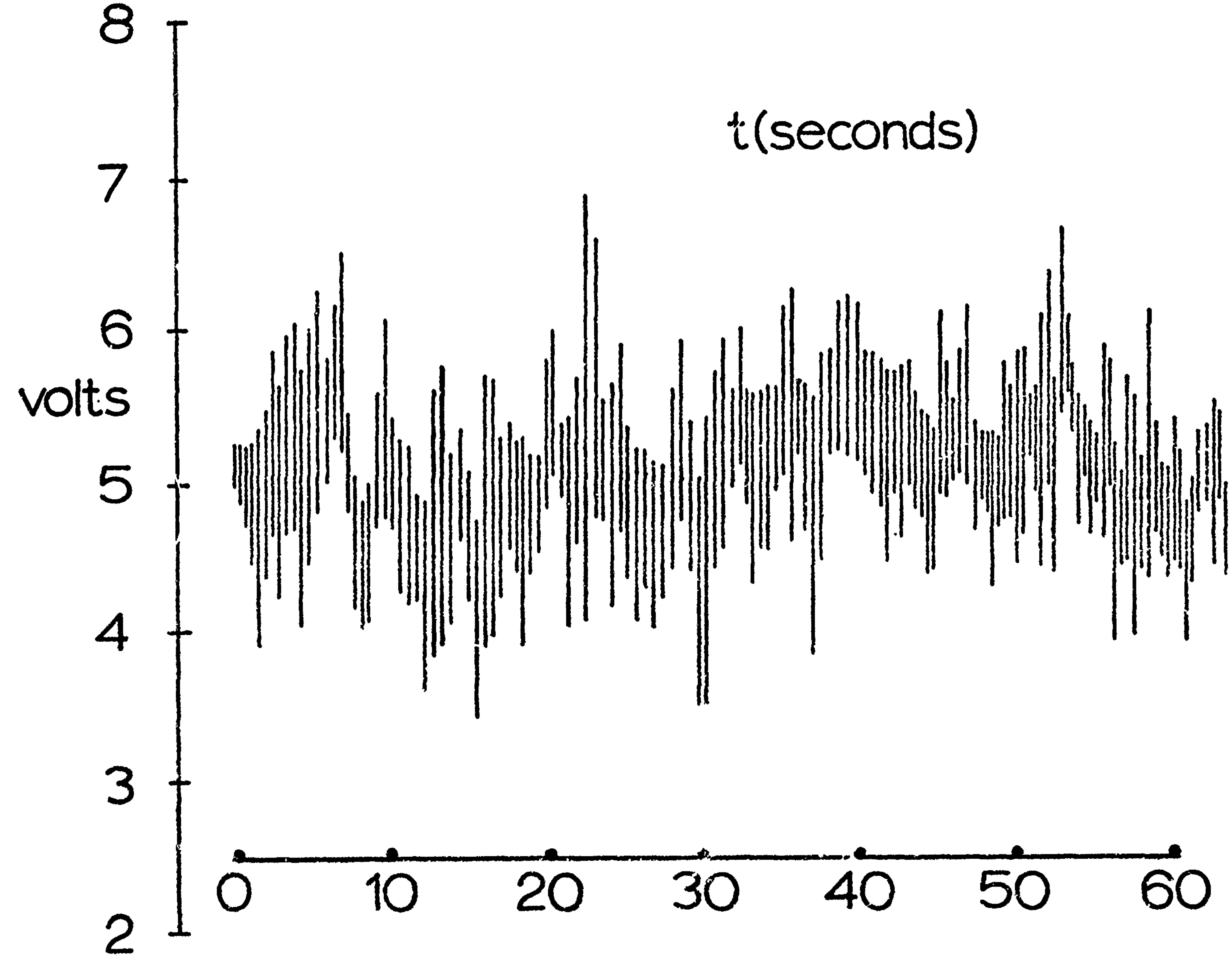


Figure 5.b

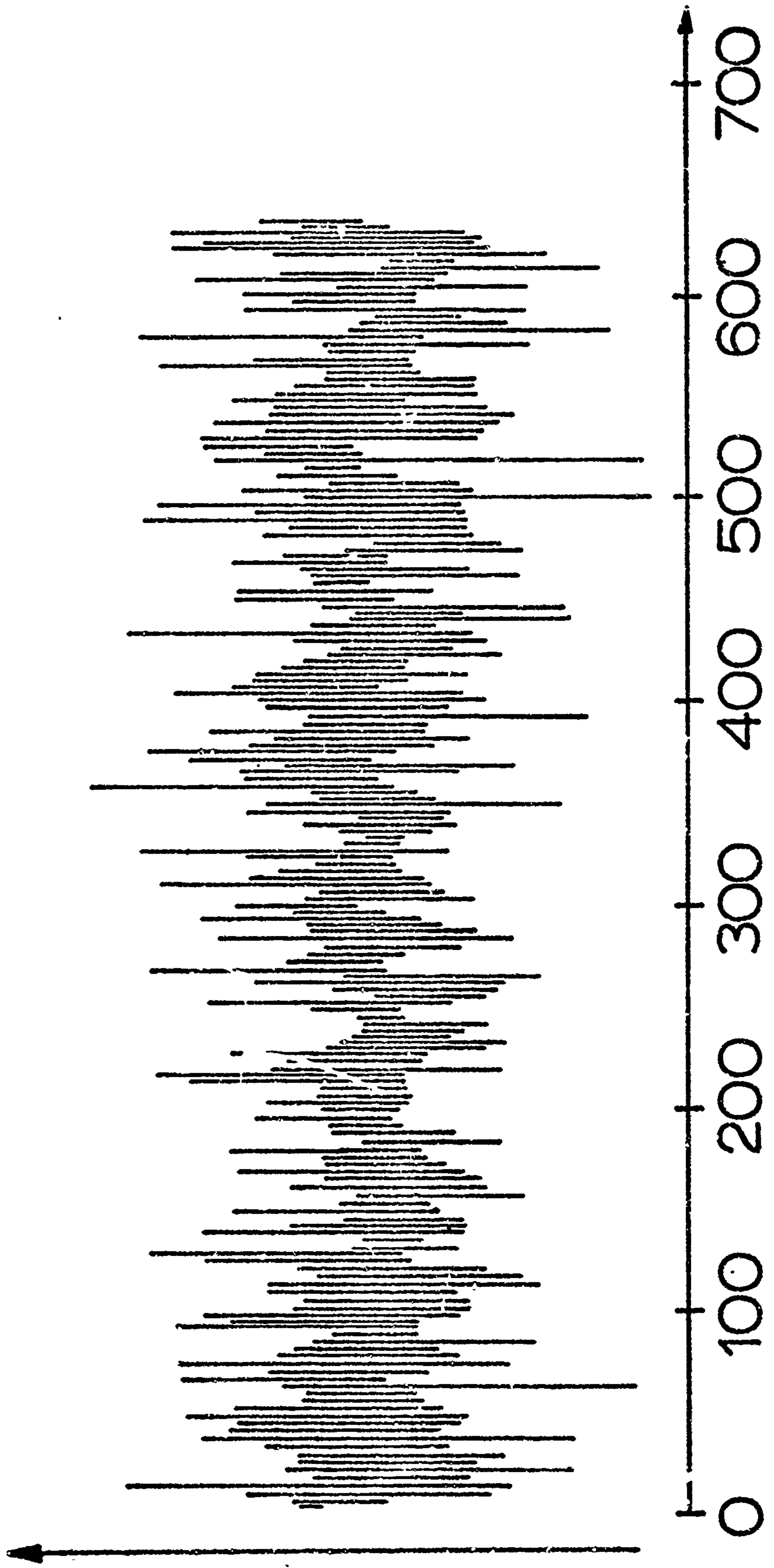


Figure 5.c

backwards through the same circuit to compensate for the shift. This doubly smoothed data was then subtracted from the original Gaussian data to form the final white noise, w(t) to be used as the input to the linear filter, H(S) (Fig. 5.d.). It is better, though not perfect. The output of the filter, x(t) [Fig. 5.e.] was then adjusted in such a way as to have a standard deviation of 25 volts and its autocorrelation function was plotted [Fig. 5.f.]. x(t) was then fed into the non-linear element, f(x) to give an output, y(t) [Fig. 5.g.]. The autocorrelation function, $\rho_y(\tau)$ was determined [Fig. 5.h.] and the histogram of y(t) plotted [Fig. 5.i.]. The hybrid computer system used in this experimentation and the method for obtaining the autocorrollographs have been described earlier [10], [11].

Considering the limited number of data points, which could be used in this hybrid system, the histogram is surprisingly good. The autocorrelation plots show that the removal of the DC-component was not as good as was hoped for. However, this discrepancy would disappear in time if a continuous Gaussian noise source were available and longer record could be used to check the autocorrelation functions.

5. Concluding Remarks.

A method has been developed to generate random time series having a predetermined autocorrelation and probability density. Needless to say that this research is still in its infancy. Also due to the lack of space we are unable to show some interesting results concerning the shape of the non-linearities and their effects to the autocorrelation function of x(t). This research will definitely continue especially to find out what are the limitations, if any, of this approach.

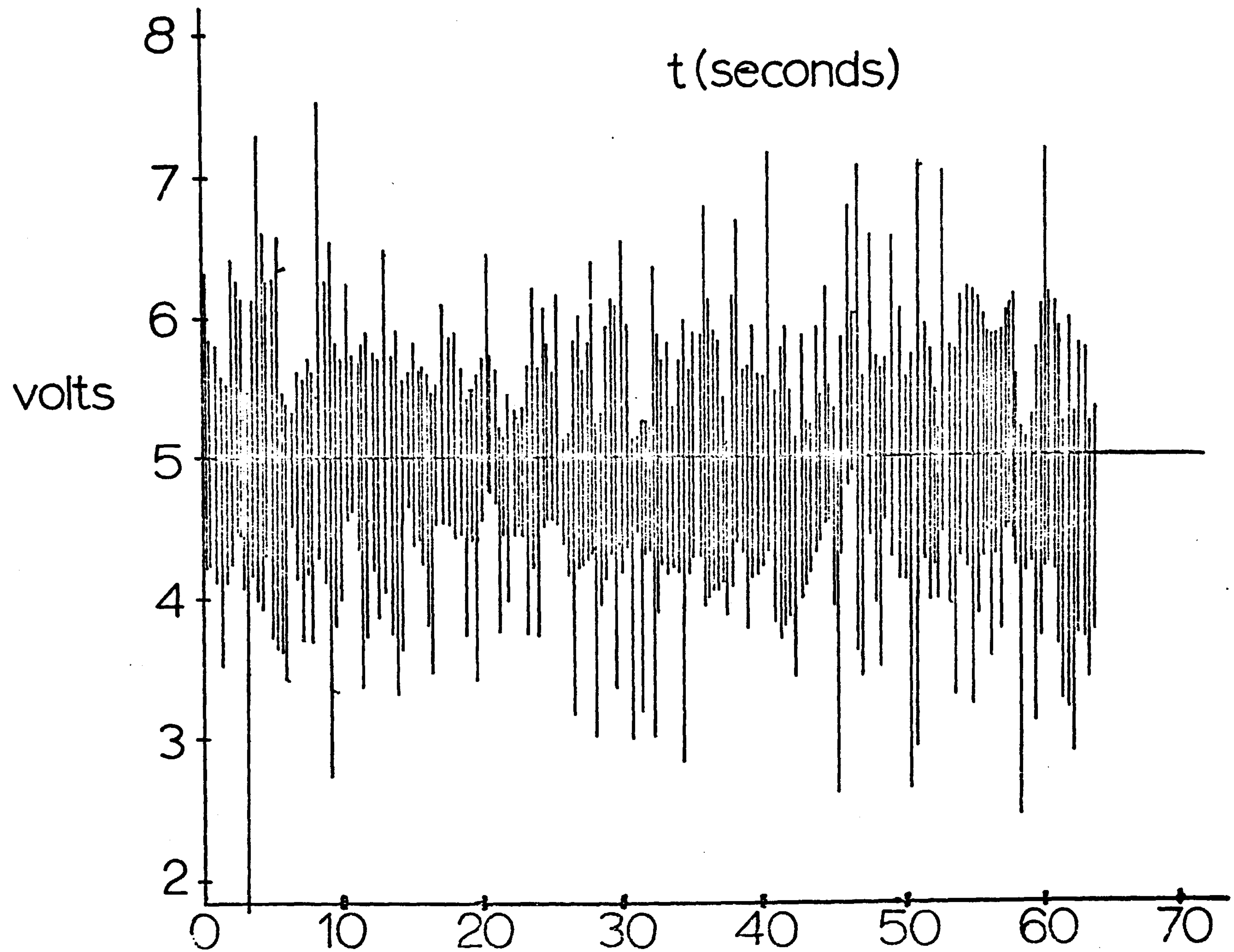


Figure 5.d

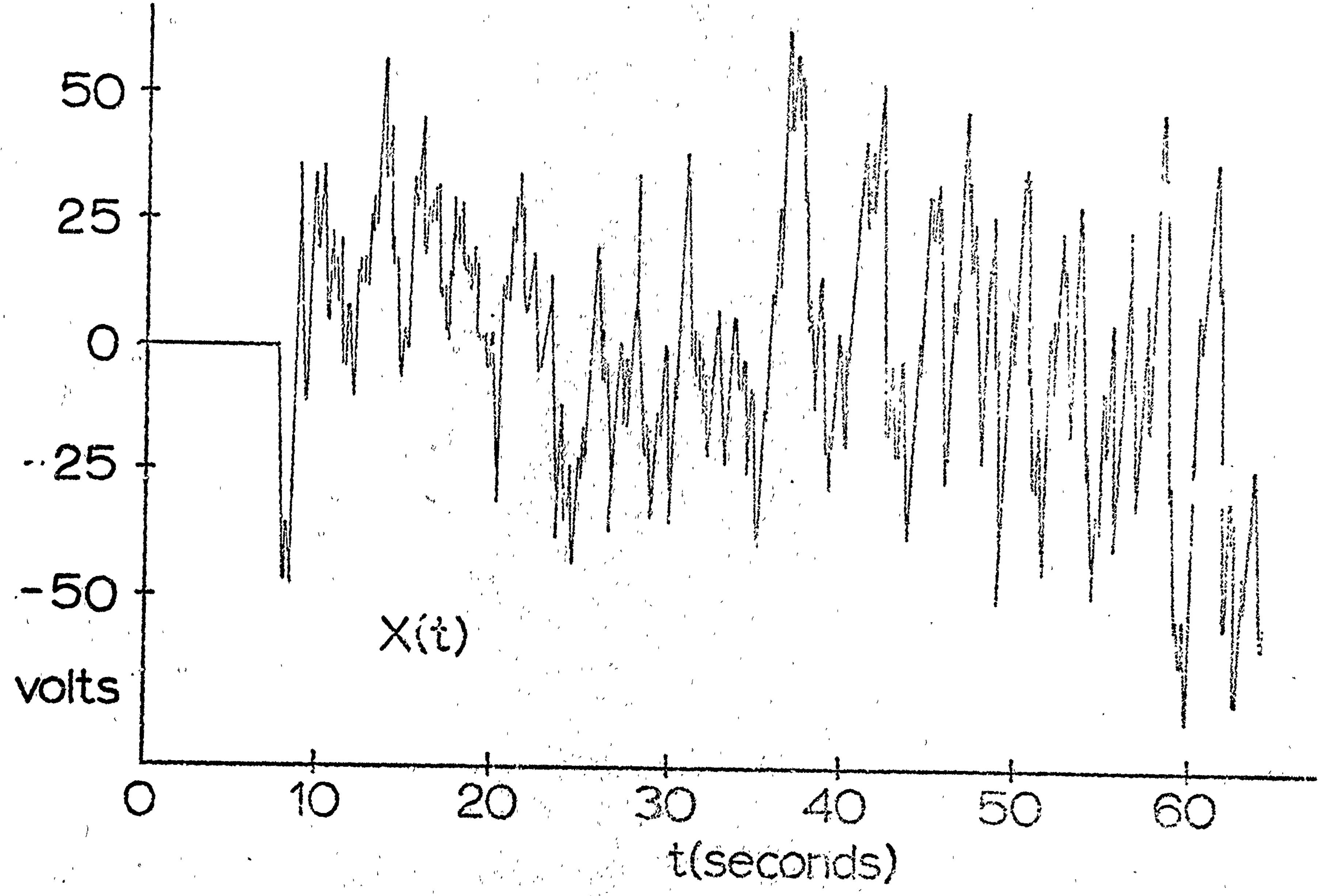


Figure 5e. The output of the linear filter $H(s)$, $x(t)$, having a normal distribution $p_x(x)$ and a predetermined autocorrelation function $\rho_x(\tau)$.

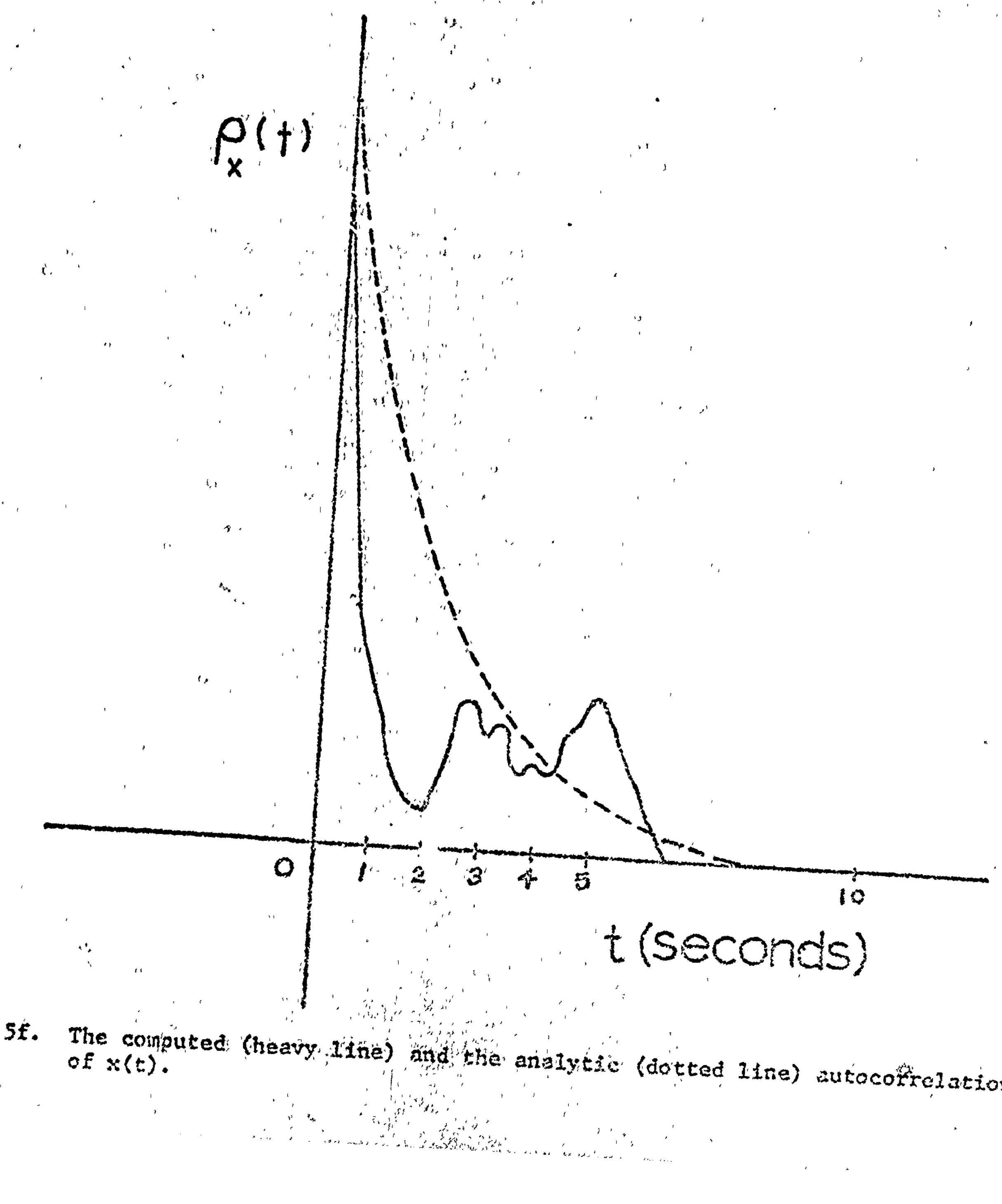


Figure 5f. The computed (heavy line) and the analytic (dotted line) autocorrelation functions of x(t).

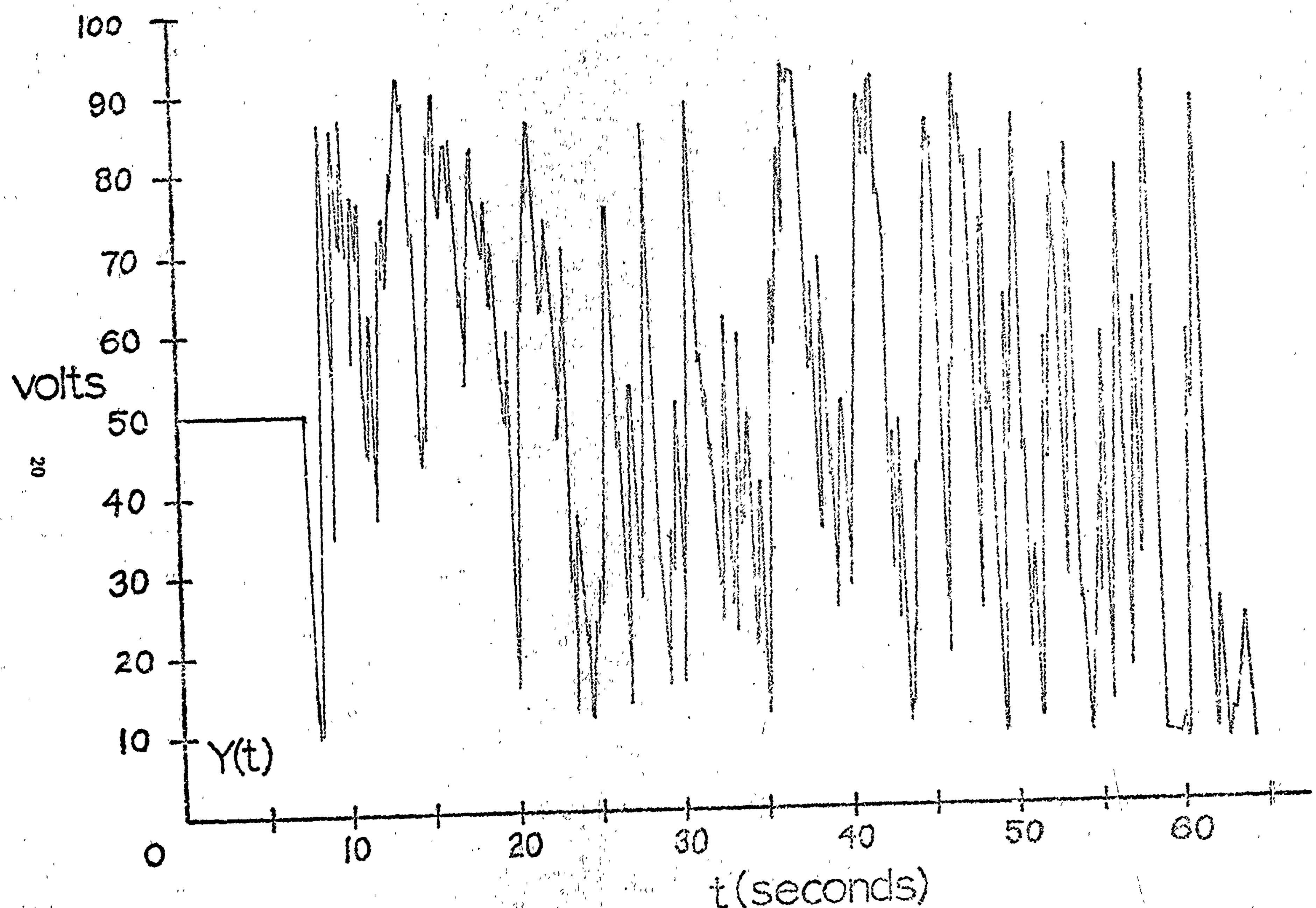


Figure 5g. The output of the nonlinear element $f(x)$, $y(t)$, having the required distribution $p_y(y)$ and autocorrelation function $\rho_y(\tau)$. (The desired simulated disturbance.)

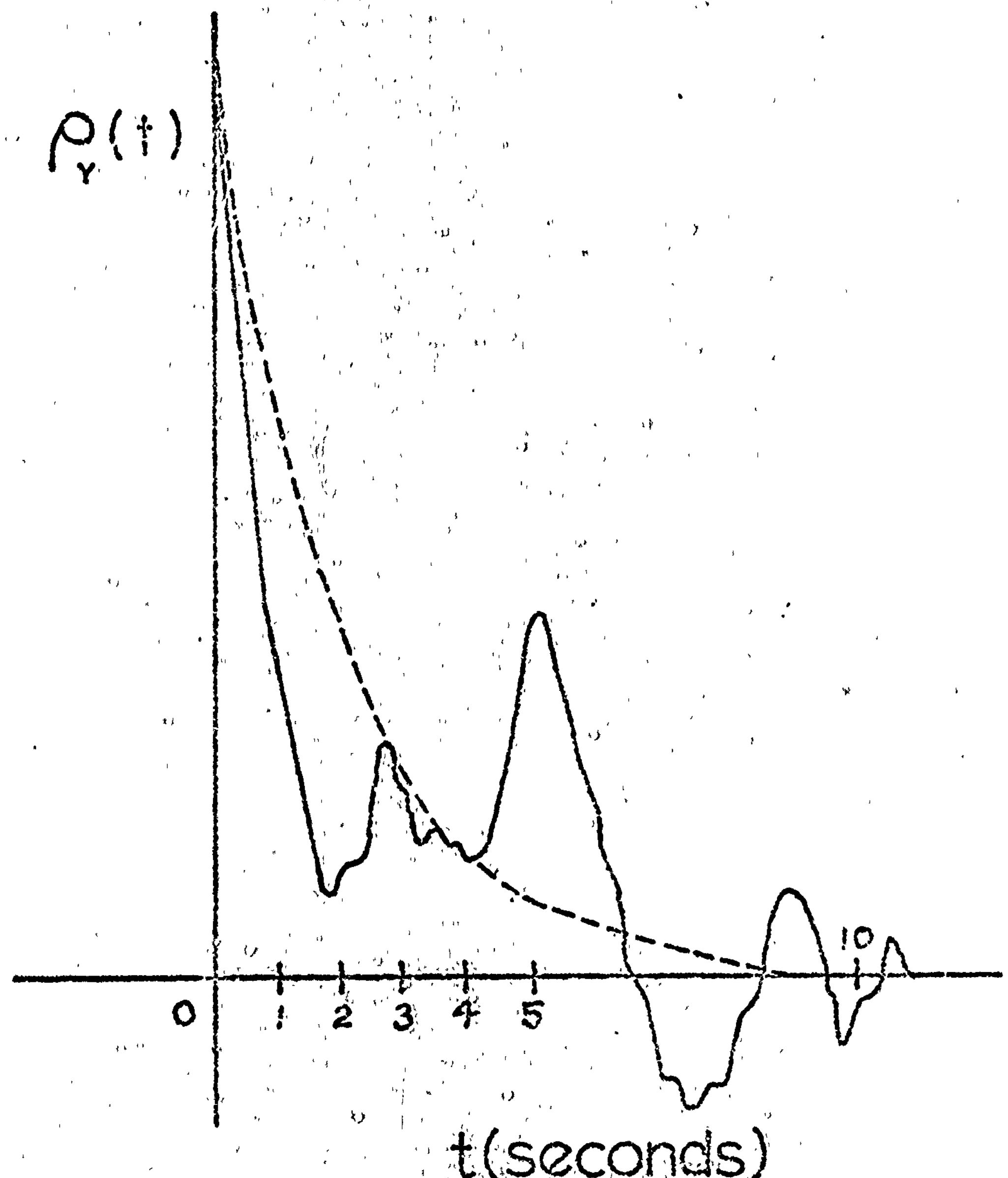


Figure 5h. The computed (heavy line) and the analytic (dotted line) autocorrelation functions of y(t).

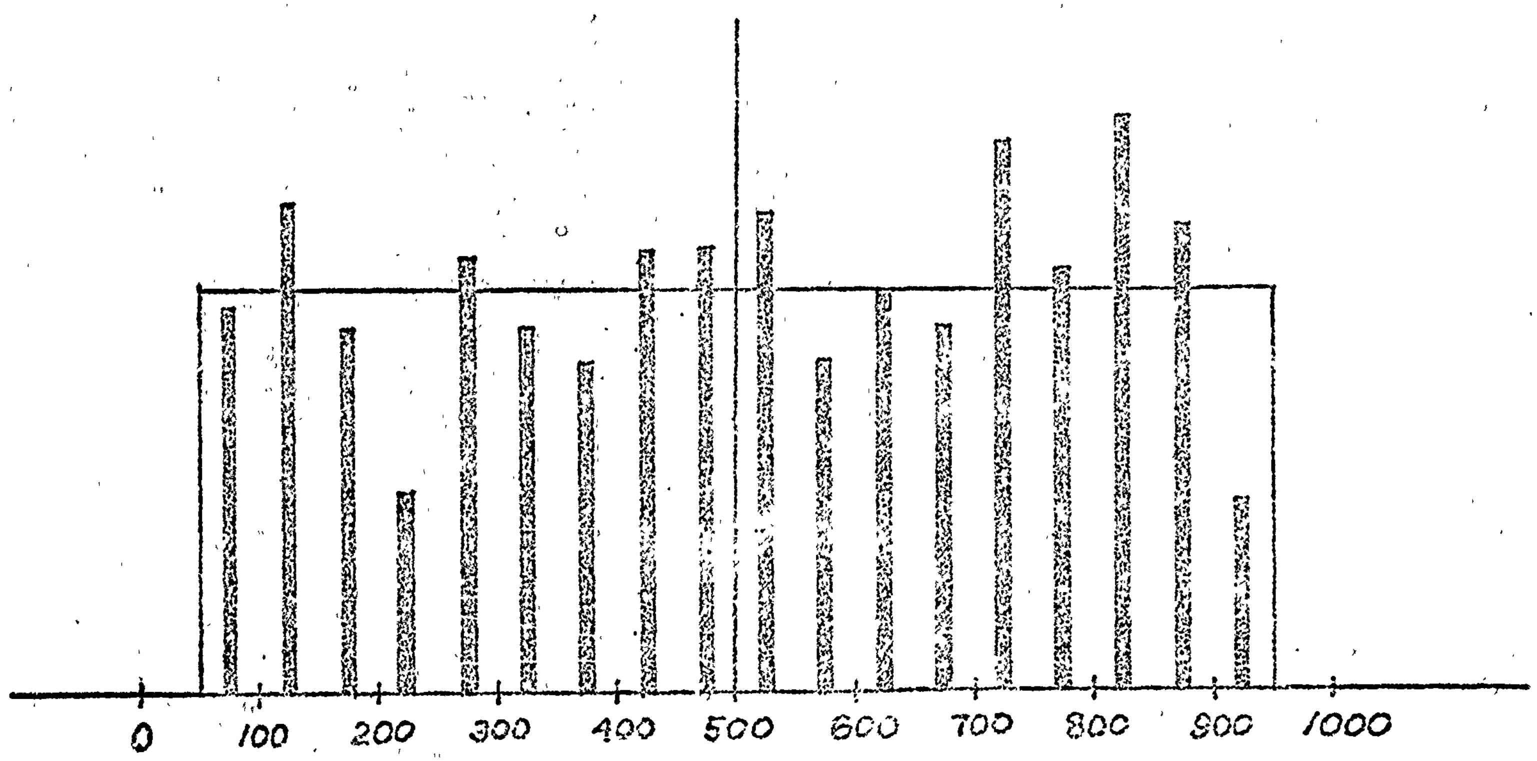
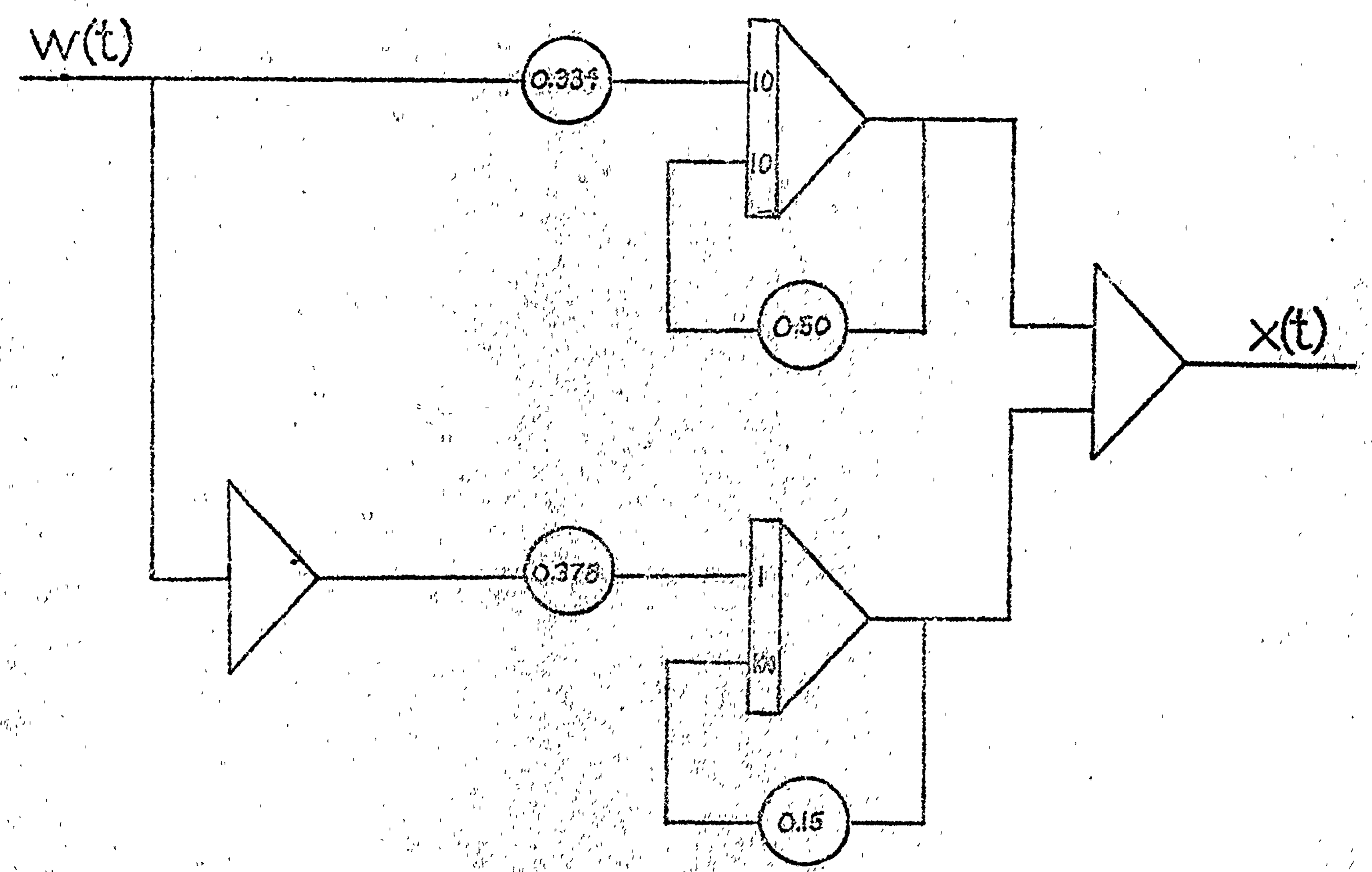


Figure 5i. The histogram of y(t) showing the computed distribution $p_y(y)$ as compared to the required uniform distribution.



Appendix 1. Figure 1. An analog computer circuit to implement H(s) for the example in Appendix 1.

Appendix No. 1.

A Simple Example.

An example, where an analytical solution can be found is given to test the validity of this method. Suppose that the output, $y(t)$ is required to have a probability density function of the form: [8]

$$p_y(y) = \frac{\alpha}{2A} \exp - \{\frac{1}{2} (\alpha^2-1)[\mathrm{erf}^{-1}(\frac{y}{2A})]^2\} , \qquad |y| \leq A$$

where erf^{-1} is the inverse of the function

$$\mathrm{erf}(x) = \int_0^x \frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}} du$$

Let the desired autocorrelation function of $y(t)$ be of the form

$$R_y(\tau) = R_y(0)\, e^{-\beta|\tau|}$$

or in normalized form:

$$\rho_y(\tau) = e^{-\beta|\tau|}$$

In this case the variance of $y(t)$ is

$$\sigma_y^2 = R_y(0) = \frac{2A^2}{\pi} \text{ arc sin } \frac{1}{1+\alpha^2}$$

For $\alpha = 1$ the density of $y(t)$ is uniform in $(-A,A)$.

$$R_y(\tau) = \frac{2A^2}{\pi} \text{ arc sin } \frac{\rho_x(\tau)}{1+\alpha^2}$$

White noise with $R_W(\tau) = \delta(\tau)$ is used as the input to the filter and the non-linear element $f(x)$ is of the form:

$$f(x) = 2A \ \operatorname{erf} \left(-\frac{x}{\alpha}\right)$$

where

$$\operatorname{erf}(x) = \int_0^x \frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}} \, du$$

Numerical example follows.

Let us assume that:

$\alpha = 1$ and $\beta = 5$ then

$$K = \operatorname{arc\,sin} \frac{1}{1+\alpha^2} = \frac{\pi}{6} = .524$$

$K_1 = 73.6$; $K_3 = 4.56$ and $K_2 = 37.5$

$B_1 = 5.0($ $B_2 = .575$ and $B_3 = .0125$

$$H(s) = .66 \left\{\frac{5.08}{s+5} - \frac{.575}{s+15} + \frac{.0125}{s+25}\right\}$$

$$= \frac{3.34}{s+5} - \frac{.378}{s+15} + \frac{.0082}{s+25}$$

Only two terms required in this case.

Consequently

$$\rho_x(\tau) = (1 + \alpha^2) \sin \frac{\pi R_y(\tau)}{2A^2}$$

which is approximately:

$$\rho_x(\tau) = (1 + \alpha^2)\left\{\frac{\pi}{2A^2} R_y(\tau) - \frac{\left[\frac{\pi}{2A^2} R_y(\tau)\right]^3}{3!} + \frac{\left[\frac{\pi}{2A^2} R_y(\tau)\right]^5}{5!}\right\}$$

These terms provide enough accuracy.

Then if we let:

$$\frac{R_y(0)\pi}{2A^2} = K = \arcsin \frac{1}{1+\alpha^2}$$

$$0 \leq K \leq \frac{\pi}{2}$$

we have:

$$\rho_x(\tau) = (1 + \alpha^2)\, K \left\{e^{-\beta|\tau|} - \frac{K^2}{6} e^{-3\beta|\tau|} + \frac{K^4}{120} e^{-5\beta|\tau|}\right\}$$

$$\phi_x(s) = (1 + \alpha^2)\, K\beta \left\{\frac{2}{(\beta^2-s^2)} - \frac{K^2}{(9\beta^2-s^2)} + \frac{K^4}{12(25\beta^2-s^2)}\right\}$$

$$= \frac{(1 + \alpha^2)\, K\beta}{12}\left\{\frac{24(9\beta^2-s^2)(25\beta^2-s^2) - 12K^2(\beta^2-s^2)(25\beta^2-s^2) + K^4(\beta^2-s^2)(9\beta^2-s^2)}{(\beta^2-s^2)\quad(9\beta^2-s^2)\quad(25\beta^2-s^2)}\right\}$$

$$= \frac{(1+\alpha^2)\,K\beta\,\{s^4[24-12K^2+K^4] - 2s^2\beta^2\,[408 - 156K^2 + 5K^4] + [5400 - 300K^2 + 9K^4]\,\beta^4\}}{12(\beta^2-s^2)\,(9\beta^2-s^2)\,(25\beta^2-s^2)}$$

This will result in the required filter as:

$$H(s) = \sqrt{\frac{(1+\alpha^2)\,K\beta}{12}} \cdot \frac{K_1\beta^2 + K_2\beta s + K_3 s^2}{(\beta+s)\,(3\beta+s)\,(5\beta+s)}$$

where

$$K_1 = \sqrt{5400 - 300\,K^2 + 9\,K^4}, \quad \text{real for } 0 \le K \le \frac{\pi}{2}$$

$$K_3 = \sqrt{24 - 12\,K^2 + K^4}, \quad \text{real for } 0 \le K \le \frac{\pi}{2}$$

$$K_2 = \sqrt{2\,\{(408 - 156\,K^2 + 5\,K^4) + K_1 K_3\}}, \quad \text{real for } 0 \le K \le \frac{\pi}{2}$$

or:

$$H(s) = \sqrt{\frac{(1+\alpha^2)\,K\beta}{12}} \left\{ \frac{B_1}{s+\beta} - \frac{B_2}{s+3\beta} + \frac{B_3}{s+5\beta} \right\}$$

where

$$B_1 = \frac{K_1 - K_2 + K_3}{8}$$

$$B_2 = \frac{K_1 - 3K_2 + 9K_3}{4}$$

$$B_3 = \frac{K_1 - 5K_2 + 25K_3}{8}$$

REFERENCES

1. R. N. McDonough and W. H. Huggins, "Best Least-Squares Representation of Signals by Exponentials," IEEE Trans. on Automatic Control, Vol. AC-13, August 1968, pp. 408-412.

2. A. Papoulis, "Probability, Random Variables, and Stochastic Processes," McGraw Hill, 1965.

3. R. Deutsch, "Non-linear Transformation of Random Processes," Prentice Hall, 1962.

4. J. B. Thomas, "Introduction to Statistical Communication Theory," John Wiley, 1969.

5. R. F. Baum, "The Correlation Function of Gaussian Noise Passed Through Non-linear Devices," IEEE Trans. on Information Theory, Vol. IT-15, July 1965, pp. 448-456.

6. R. F. Baum, "The Correlation Function of Smoothly Limited Gaussian Noise," IRE Trans. on Information Theory, Vol. IT-3, September 1957, pp. 193-197.

7. P. I. Kuznetsov, R. L. Stratonovich, and V. I. Tikhonov," Non-linear Transformations of Stochastic Processes," Pergamon Press, 1965.

8. M. Abramowitz and I. Stegun (Eds.), "Handbook of Mathematical Functions," National Bureau of Standards, Appl. Math. Series 55, 1964.

9. W. H. Kautz, "Transient Analysis in the time domain," IRE Trans. on Circuit Theory, Vol. CT-1, September 1956, pp. 29-39.

10. P. E. Valisalo, "A Unique Systems Engineering Laboratory for Engineering Education and Research," Simulation, Vol. 11, No 4, October, 1968.

11. M. Cuenod, A. Durling and P. Valisalo, "Analysis of Random Processes on Hybrid Computers," Proceedings of the 5th International Congress of A.I.C.A., Vol. I., 1968.

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of Florida	2a. REPORT SECURITY CLASSIFICATION Unclassified	
	2b. GROUP Unclassified	
3. REPORT TITLE "Generation of Random Time-series through Hybrid Computation"		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)		
5. AUTHOR(S) (Last name, first name, initial) P. E. Valisalo and A. H. Haddad		
6. REPORT DATE May 1970	7a. TOTAL NO. OF PAGES 28	7b. NO. OF REFS 11
8a. CONTRACT OR GRANT NO. DAH C04 68C0002	9a. ORIGINATOR'S REPORT NUMBER(S) Technical Report No. 35	
b. PROJECT NO. 2T0 14501B81B		
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.		
10. AVAILABILITY/LIMITATION NOTICES This document has been approved for public release and sale; its distribution is unlimited.		
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY ARO-Durham	

13. ABSTRACT

In many engineering design problems it is possible to collect data of the environmental disturbances which are acting upon ~~our~~ the systems. This data can be analyzed by determining its autocorrelation and probability density function. When seeking solutions through simulation it is desirable to be able to generate random time series having a predetermined autocorrelation and probability density. This paper describes a method to control both simultaneously.

The proposed system is composed of a linear filter, H(s), with white Gaussian noise as input, followed by a nonlinear element f(x), where x(t) is the output of H(s). The output of the system y(t) is required to have a predetermined probability density $p_y(y)$ and a predetermined normalized autocorrelation function $\rho_y(\tau)$. The nonlinearity f(x) is designed to give the required density $p_y(y)$, and is relatively easy to design using the relationship

$$f(x) = F_y^{-1}[F_x(x)]$$

DD FORM 1 JAN 64 1473

Security Classification

Abstract (continued)

where F_y^{-1} is the inverse of $F_y(y)$ which is the cumulative distribution function of y, and

$$f_x(x) = \int_{-\infty}^{x} \frac{e^{-\frac{\xi^2}{2}}}{\sqrt{2\pi}} d\xi$$

is the cumulative Gaussian distribution.

The nonlinearity f(x) may be designed manually or in a digital computer by using a double table look up or in a hybrid computer system. It can then be stored in a variable diode function generator of an analog computer.

This nonlinear element, however, changes the autocorrelation of the input, x(t). The amount of change can be determined by

$$\rho_y(\tau) = \sum_{n=1}^{N} \frac{K_n^2}{n!} [\rho_x(\tau)]^n$$

where

$$K_n = \int_{-\infty}^{\infty} f(x)H_n(x) \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx \qquad \text{and}$$

$H_n(x)$ is the Hermite polynomial of order n.

This calculation can be done most conveniently in a hybrid computer system.

The normalized autocorrelation of the intermediate stage, x(t), $\rho_x(\tau)$ must now be approximated by a sum of exponentials:

$$\hat{\rho}_x(\tau) = a_1e^{-s_1|\tau|} + a_2e^{-s_2|\tau|} + \ldots$$

by using a hybrid computer system. One approach which may be used to determine the a_i's and s_i's is discussed by McDonough and Huggins. The filter H(s) may be obtained from the a_i's and s_i's by spectral factorization.

By forcing the system with white Gaussian noise the system output, y(t) will have the desired autocorrelation function and probability density.